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Analytical Solution for Phase Modulation in BURST Imaging with Optimum Sensitivity

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In order to improve the efficiency of BURST imaging, a new phase-modulation scheme of the DANTE-type RF excitation is proposed. It is shown that analytical optimization of the phase-modulation scheme with optimal SNR can be found for an arbitrary number N of subpulses in the RF excitation. Both theory and experiment indicate a maximum attainable gain in efficiency of the square root of N, compared to nonmodulated (constant-phase) excitation schemes. © 1995 Academic Press, Inc.

INTRODUCTION

BURST imaging is a class of MR imaging techniques which allows fast, single-shot imaging on standard clinical scanners, using a DANTE pulse train for excitation. A disadvantage of this technique is the low SNR, which can be partially overcome by phase modulation of the RF pulses. Here, an analytical solution for this phase modulation is presented with optimum SNR, for any number of pulses, and is experimentally verified.

The original technique, proposed by Hennig and Mueri (*1*), uses a train of RF pulses in rapid succession (hence called a BURST pulse) in combination with a continuous B_0 gradient. Refocusing of the gradient generates a train of echo signals. Each of these echoes is phase encoded, allowing collection of a 2D image within a single repetition. Note that since only a fraction (a strip) of each imaging voxel within the sample magnetization is excited, corresponding to a single frequency band of the DANTE (*2*) excitation, the magnetization between these strips is left unused, resulting in low sensitivity. Furthermore, fast repetition of the pulse sequence compared to $1/T_1$ results in saturation effects.

For fast 3D BURST imaging, the SNR can be improved by moving the location of the strips within a voxel on successive repetitions (FS-BURST (*3, 4*)). An alternative way to make more efficient use of the magnetization within the object is phase modulation of the DANTE excitation, as originally proposed by Bodenhausen *et al.* (*2*), and introduced to BURST imaging by Le Roux *et al.* (*5*) and Lowe *et al.* (*6, 7*). This approach, in the following referred to as phase-modulated (PM) BURST, increases the fraction of excited magnetization within each voxel.

PM BURST excitation schemes for improved SNR aim at increasing the echo amplitude without degradation of the echo envelope. Several different modulation strategies can be used, and specific phase schemes have been calculated by computer optimization (*5, 7*). Here, we present an analytical solution resulting in optimum SNR and uniform echo envelope.

THEORY

In the following it is assumed that (complex transverse) magnetization pattern is the Fourier transform of the BURST excitation. This requires a constant and uniform field gradient, infinite relaxation times T_1 and T_2, negligible motion and diffusion effects, and small flip angles (linear response theory). Under the same assumptions, the time course of the signal generated by refocusing the gradient corresponds to the inverse Fourier transform of the magnetization pattern. For simplicity, in the following, no distinction has been made between the forward and the inverse transforms.

Two conditions are imposed on the solution for the modulation scheme: (1) the spacing of the echoes should be unchanged (i.e., no echoes should drop out as result of the modulation), and (2) the amplitude should be uniform. Given these conditions, one can maximize the signal amplitude for a given number of strips. No conditions are imposed on the phase of the echoes, since the phase of the echoes can be corrected in postprocessing. Finding the maximum signal therefore means that the modulus of the (complex) amplitude ($|A_k|$) of the echoes should be maximal and equal for all echoes. Equivalently, the power spectrum ($|A_k|^2$) of the magnetization should be uniform.

The problem of finding the optimal phase modulation, given the conditions stated above can be solved by considering the Fourier transform of the echo train, i.e., the complex magnetization profile. The echo train can be considered as the convolution of a train of delta pulses with the profile of

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a single echo, multiplied by an envelope function. The Fourier transform is therefore a product of a periodic delta train and the image profile of the object in the magnet, convoluted with the transform of the envelope function (see Fig. 1). Phase modulation does not change the image profile nor the envelope function, so these are not considered here. In the following, one period of the delta train in the magnetization profile is calculated, corresponding to the strip(s) in one image voxel.

Phase modulation of the RF pulses leads to excitation of several (N) strips within each imaging voxel, each with its own amplitude ($|a_n|$) and phase (ϕ_n). It will first be shown that the requirements for the echo amplitude (A_k) translate to conditions of the autocorrelation function of the magnetization profile. Then the consequences of these conditions will be shown and a solution will be presented.

The autocorrelation function (ac_q) is given by

$$\mathrm{ac}_q = \sum_{n=0}^{N-1} a_n^* a_{n+q}. \qquad [1]$$

A property of the autocorrelation is that its FT corresponds to the power spectrum (*8*):

$$\begin{aligned} |A_k|^2 &= \mathrm{FT}(a_n)^*\mathrm{FT}(a_m) \\ &= \sum_{n=0}^{N-1} a_n^* \exp(i2\pi kn/N) \sum_{m=0}^{N-1} a_m \exp(-i2\pi km/N) \\ &= \sum_{q=0}^{N-1} \mathrm{ac}_q \exp(-i2\pi kq/N) \\ &= \mathrm{FT}(\mathrm{ac}_q). \end{aligned} \qquad [2]$$

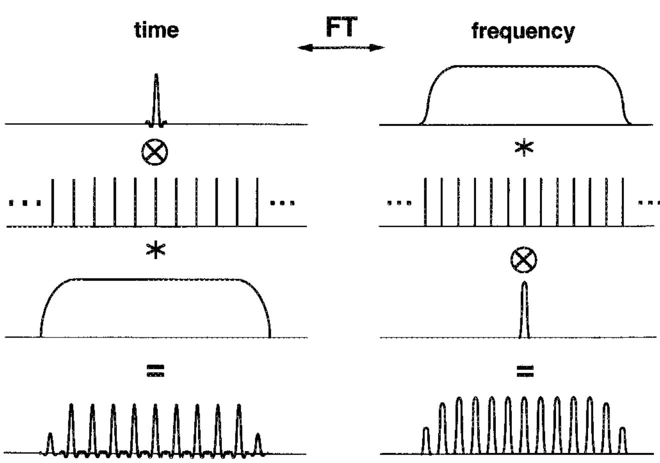

FIG. 1. The BURST excitation and its Fourier transform, the magnetization profile. The calculated amplitude and phase of the strips (a_n) correspond to one period of the delta train on right-hand side of the second line.

Using Eq. [2], the condition of constant amplitude for all the echoes translates to the condition that the autocorrelation is zero for all shifts q except for q equal to zero:

$$\begin{aligned} |A_k| = \text{constant} &\Rightarrow |A_k|^2 \\ &= \text{constant} \Rightarrow \mathrm{ac}_q = 0; \quad q \neq 0. \end{aligned} \qquad [3]$$

The maximum gain can also be derived from Eq. [2]. Since all the echoes have the same amplitude, it suffices to calculate only a single one, for example, A_0:

$$\begin{aligned} |A_0|^2 &= \sum_{q=0}^{N-1} \mathrm{ac}_q = \mathrm{ac}_0 \\ &= \sum_{n=0}^{N-1} a_n^* a_n \Rightarrow \begin{cases} |a_n| = 1; & \text{for max } |A_0| \\ |A_0|^2 = N \Rightarrow |A_0| = \sqrt{N}. \end{cases} \end{aligned} \qquad [4]$$

This shows that the amplitude of the magnetization of all the excited strips should be unity (maximum magnetization) and the maximum gain in signal will be $\sqrt{N}$, the square root of the number of strips per voxel.

As the amplitude of the transverse magnetization of all the strips equals one, only their phases (ϕ_n) need to be calculated. The condition for the autocorrelation leads to

$$\begin{aligned} \mathrm{ac}_q = 0 &\Rightarrow \sum_{n=0}^{N-1} a_n^* a_{n+q} \\ &= \sum_{n=0}^{N-1} \exp(i(\phi_{n+q} - \phi_n)) = 0; \quad q \neq 0. \end{aligned} \qquad [5]$$

This can be solved for any N by

$$\phi_n = \frac{2\pi}{N} \frac{n(n+p)}{2}; \quad p \in \mathbb{Z}, \qquad [6]$$

where p is any integer number. This solution can be verified by substitution of Eq. [6] in Eq. [5]. The resulting autocorrelation can be written as

$$\begin{aligned} \mathrm{ac}_q &= \sum_{n=0}^{N-1} \exp(i(\phi_{n+q} - \phi_n)) \\ &= \sum_{n=0}^{N-1} \exp\left[i \frac{2\pi}{N} \frac{1}{2} ((n+q)(n+q+p) - n(n+p)) \right] \\ &= \sum_{n=0}^{N-1} \exp\left[i \frac{2\pi}{N} \frac{1}{2} (n(n+p) \right. \\ &\qquad \left. + 2nq + q(q+p) - n(n+p)) \right] \\ &= \exp\left(i \frac{2\pi}{N} \frac{1}{2} q(q+p) \right) \sum_{n=0}^{N-1} \exp\left(i \frac{2\pi}{N} \frac{1}{2} nq \right) \\ &= \begin{cases} (-1)^{(N+p)} N; & q = kN \\ 0; & q \neq kN \end{cases} \quad k \in \mathbb{N}. \end{aligned} \qquad [7]$$

Therefore, it becomes obvious that the condition from Eq. [3] is satisfied for any N and p. With this solution, the different components $[\exp(i(\phi_{n+q} - \phi_n))]$ contributing to a certain autocorrelation (say ac_1) are evenly distributed over a unit circle in the complex plane, thereby canceling to zero (see Fig. 2).

The period of the resulting phase modulation follows from

$$\phi_N = \frac{2\pi}{N}\frac{N(N+p)}{2} = \pi(N+p) = \phi_0;$$
$$(N, p \in \mathbb{O}) \vee (N, p \in \mathbb{E})$$
$$\phi_{2N} = \frac{2\pi}{N}\frac{2N(2N+p)}{2} = 2\pi(2N+p) = \phi_0;$$
$$N, p \in \mathbb{Z}, \qquad [8]$$

where $\mathbb{O}$ is the set of odd numbers and $\mathbb{E}$ the set of even numbers. Thus for $N + p$ even, the phase is repeated every N strips. For $N + p$ odd the period becomes $2N$ strips. In this case the magnetization is not repeated every voxel, but only over every two voxels. This would suggest that the number of echoes is increased by a factor of two. However, the autocorrelation is nonzero for $q = N$ (Eq. [9]):

$$\begin{aligned} ac_N &= \sum_{n=0}^{N-1} \exp\left[i\frac{2\pi}{N}\frac{1}{2}((n+N)(n+N+p) - n(n+p))\right] \\ &= \sum_{n=0}^{N-1} \exp\left[i\frac{2\pi}{N}\frac{1}{2}(n(n+p) + N(N+p)\right. \\ &\quad \left. + 2nN - n(n+p))\right] \\ &= \exp(i\pi(N+p)) \sum_{n=0}^{N-1} \exp\left(i\frac{2\pi}{N}nN\right) = (-1)^{N+p}N. \end{aligned} \qquad [9]$$

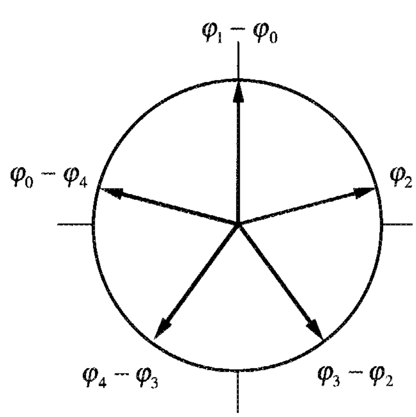

FIG. 2. The different components of the autocorrelation of the phase of the magnetization pattern with $N = 5$ for shift q equal 1.

Therefore amplitudes (A_k) of half of the resulting echoes are zero, and the resulting number of echoes is not changed.

The phase of the pulses (and echoes) can now be calculated by Fourier transformation of the magnetization profile:

$$\begin{aligned} A_k &= \sum_{n=0}^{N-1} \exp\left[i\frac{2\pi}{N}\left(\frac{1}{2}n(n+p) - kn\right)\right] \\ &= \sum_{n=0}^{N-1} \exp\left[i\frac{2\pi}{N}\left(\frac{1}{2}(n+k)(n+k+p) - k(n+k)\right)\right] \\ &= \sum_{n=0}^{N-1} \exp\left\{i\frac{2\pi}{N}\left[\frac{1}{2}(n(n+p) + k(k+p)\right.\right. \\ &\quad \left.\left. + 2kn) - kn - k^2\right]\right\} \\ &= \sum_{n=0}^{N-1} \exp\left[i\frac{2\pi}{N}\frac{1}{2}(n(n+p) - k(k-p))\right] \\ &= \exp\left(-i\frac{2\pi}{N}\frac{1}{2}k(k-p)\right)\sum_{n=0}^{N-1}\exp\left(i\frac{2\pi}{N}\frac{1}{2}n(n+p)\right) \\ &= \exp\left(-i\frac{2\pi}{N}\frac{1}{2}k(k-p)\right)A_0. \end{aligned} \qquad [10]$$

This shows that the phases of the RF pulses and the echoes follow essentially the same formula as the solution for the magnetization (Eq. [6]), apart from an (arbitrary) overall phase (included in A_0). It can be shown in a very similar fashion that the solutions with period $2N$ lead to equivalent patterns of echo phase.

METHODS

Phantom experiments were performed on a 4.7 T GE/OMEGA animal scanner (General Electric), equipped with 0.2T/m, actively shielded, gradients. A standard Helmholtz RF coil was used for both transmit and receive. After BURST excitation, the gradient was inverted twice, and the echo signals were collected following the second inversion. This was done to eliminate distortion of the echo-train envelope due to T_2^* effects (*3*). To investigate the attainable SNR improvement with the proposed modulation scheme of Eq. [6], N was varied between 1 and 64 (steps of 1), and the phases were calculated from Eq. [6] with $p = 1$. At $N = 1$, the pulse power was adjusted to achieve a total nutation angle (sum of flip angles of all subpulses) of just under 90°, and increased with $\sqrt{N}$ for all other experiments. To investigate the magnetization profile resulting from a BURST excitation, the echo train was Fourier transformed.

RESULTS

The results of the measurements on SNR as a function of the number of phase-modulation steps (N) in the BURST

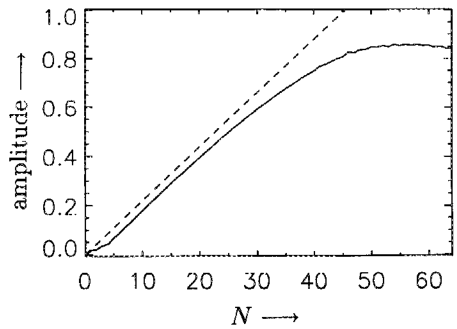

FIG. 3. The square of the total intensity of the echo train, as a function of the number of strips per voxel N. The straight line indicates the theoretical dependence. The vertical scale is arbitrary.

excitation are summarized in Fig. 3. The square of the amplitude of the echo-train envelope is displayed as a function of N, together with the theoretical curve. For $N < 32$, the experimental curve closely follows the theoretical curve, whereas for $32 < N < 64$, the increase of signal amplitude with N is slower than predicted by the theory and levels off above $N = 60$. This is explained by nonideal excitation and diffusion effects, both distorting the strip profile. As the number of strips in one voxel approaches the number of echoes, the gain in signal starts to drop because of overlap of neighboring strips. Figure 4 shows some examples of magnetization profiles for different N. Clearly illustrated is the increase in the number of strips with increasing N. The effects on the phase of the echoes is demonstrated in Fig. 5, showing the phase of the standard BURST ($N = 1$), modulation with $N = 5$, and the phase after correction using Eq. [10]. This shows that the phase can be recovered in postprocessing. The zero-order phase is not calculated in Eq. [10], as discussed under Theory, and therefore not corrected.

An interesting analogy to the solution for optimum phase modulation presented above is the one with the frequency-shifted (FS) BURST technique. In this technique, a constant-phase BURST excitation is used, resulting in excitation of a single strip per voxel. However, on successive repetitions, the excitation frequency is shifted by a small amount, resulting in an effective shift of each strip through a voxel. Thus with every excitation, new strips are created which contain magnetization untouched by previous BURST excitations, leading to a reduction of saturation effects and improved SNR. When this principle is applied within a single BURST excitation, that is, by shifting the frequency during the RF pulse train, the same expression can be found for the phase scheme of the pulses.

CONCLUSIONS

We have presented an analytical solution for phase-modulation schemes of BURST excitation, leading to optimal SNR. The solution is simple and straightforward, yields a uniform gain in echo amplitude, and has been experimentally

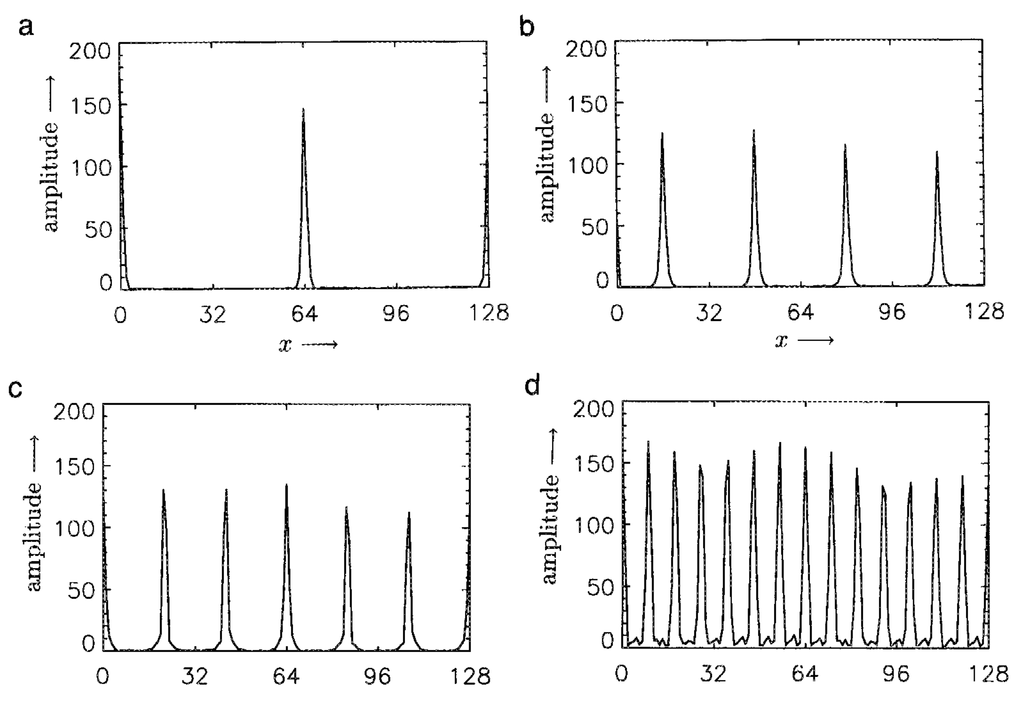

FIG. 4. Examples of the resulting magnetization profiles as a function of location x for (a) $N = 1$, (b) $N = 2$, (c) $N = 3$, and (d) $N = 7$. The plots show a section of the magnitude spectrum of the echo train, corresponding to a total width of two voxels.

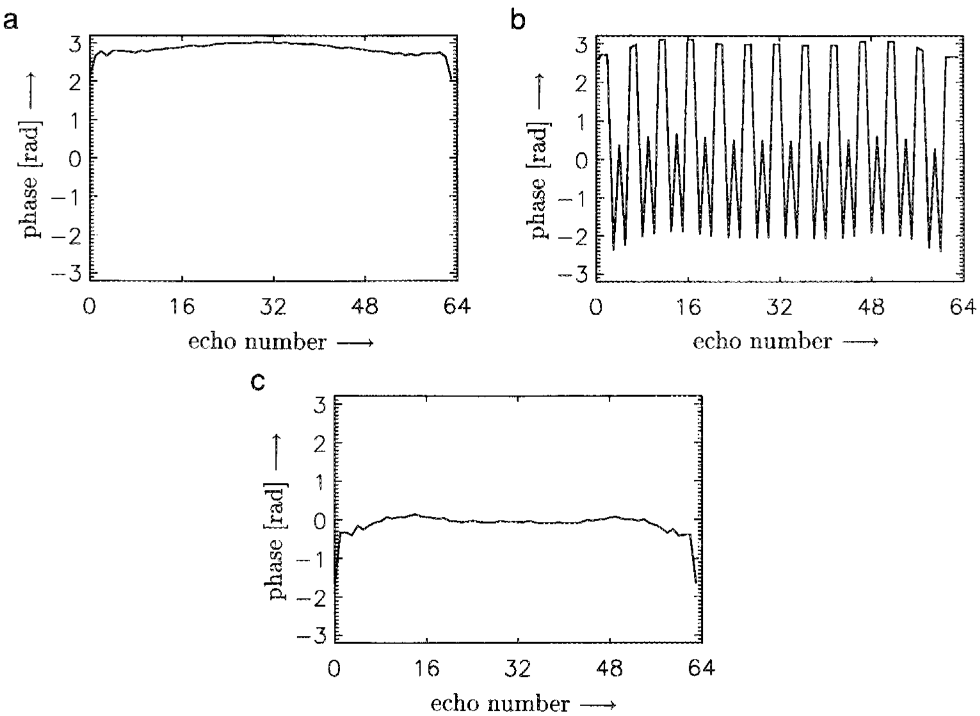

FIG. 5. An example showing the effects on the phase of the echoes as a function of echo number: (a) $N = 1$, (b) $N = 5$, and (c), $N = 5$ corrected according to Eq. [10].

verified. Although theoretical predictions indicate a SNR gain of $\sqrt{N}$ compared to single-phase BURST, the practical gain is limited by imperfections in the excitation, attributed to relaxation and diffusion effects.

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